

LORENTZ VIOLATION AT ONE LOOP

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The proof of one-loop renormalizability of the general Lorentz- and CPT-violating extension of quantum electrodynamics is described. Application of the renormalization-group method is discussed and implications for theory and experiment are considered.

1 Introduction

Invariance of the standard model under both Lorentz and CPT transformations is confirmed to high precision by experiment.¹ However, it is still possible that these symmetries may be violated in nature by interactions which are too small to be observable by current experiments. A framework exists for the description of such effects in terms of a small perturbation to the standard model at low energies called the standard-model extension.² This theory, based on a lagrangian which is observer Lorentz covariant and is constructed from the fields of the standard model, forms the basis for much of the work described in this volume and its parameters have been bounded by various experiments.¹ It is considered to be the low energy limit of some more complete theory, valid at the Planck scale, such as noncommutative field theory³ or string theory.⁴

It is of interest to ask how the low energy quantum field theory relates to the underlying theory at high energies. Studies of microcausality and stability in the standard model extension⁵ suggest that nonrenormalizable terms play an essential role in the theory at energies approaching the Planck scale. The aim of this talk is to investigate the relationship between the low and high energy theories from a different perspective. I will describe the results of applying the renormalization-group method^{6,7} to the general Lorentz- and CPT-violating extension of QED. Although some work has been done to calculate one-loop contributions to this theory, it is incomplete and so the full one-loop analysis,⁸ including a proof of renormalizability at one-loop, a generalization of the Furry theorem⁹ and the calculation of all one-loop divergences will be described here.

The renormalization-group method is used to study a quantum field theory over a wide range of energies. In particular, the running of the parameters in the theory over the given energy range must be found by calculating the relevant beta functions, usually perturbatively. Such is the importance of the beta function in conventional quantum field theory that it is now known up to three loops for a general gauge field theory.¹⁰ The calculation of the beta functions for the parameters which describe Lorentz- and CPT-violation is

therefore a significant step in any attempt to understand the standard-model extension and its relation to any underlying theory. I will describe how the one-loop beta functions have been used to find the running of the parameters in the theory and the implications for experiment and theory will be considered.

2 Framework

The lagrangian \mathcal{L} for a fermion field ψ of mass m is ²

$$\mathcal{L} = \frac{1}{2}i\bar{\psi}\Gamma^\mu \overleftrightarrow{D}_\mu \psi - \bar{\psi}M\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \frac{1}{2}(k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu}, \quad (1)$$

where $\Gamma^\nu = \gamma^\nu + \Gamma_1^\nu$ and $M = m + M_1$, with

$$\begin{aligned} \Gamma_1^\nu &\equiv c^{\mu\nu}\gamma_\mu + d^{\mu\nu}\gamma_5\gamma_\mu + e^\nu + if^\nu\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}\sigma_{\lambda\mu}, \\ M_1 &\equiv a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}. \end{aligned} \quad (2)$$

The coefficients $a_\mu, b_\mu, c_{\mu\nu}, d_{\mu\nu}, e_\mu, f_\mu, g_{\lambda\mu\nu}, H_{\mu\nu}, (k_{AF})_\mu$ and $(k_F)_{\kappa\lambda\mu\nu}$ control Lorentz violation and are real because \mathcal{L} is hermitian. Of these, $a_\mu, b_\mu, e_\mu, f_\mu, g_{\lambda\mu\nu}$ and $(k_{AF})_\mu$ control CPT violation, M_1 and $(k_{AF})_\mu$ have dimensions of mass, and Γ_1^μ and $(k_F)_{\kappa\lambda\mu\nu}$ are dimensionless. $c_{\mu\nu}$ and $d_{\mu\nu}$ are traceless, $H_{\mu\nu}$ and $g_{\lambda\mu\nu}$ are antisymmetric on their first two indices, $(k_F)_{\kappa\lambda\mu\nu}$ has the symmetries of the Riemann tensor and $(k_F)_{\mu\nu}{}^{\mu\nu} = 0$.

The lagrangian in (1) is observer Lorentz invariant by construction,² but it is not invariant under particle Lorentz transformations which leave the coefficients for Lorentz violation unchanged. Inertial frames exist where, as a result of their observer Lorentz dependence, these coefficients are very large and a perturbative expansion in them is inappropriate. This work is restricted to so-called concordant frames,⁵ where the parameters that determine the Lorentz violation are extremely small, such as in the Earth frame. In this context, a one-loop diagram is therefore defined to contain exactly one closed loop and to be at most first order in the coefficients for Lorentz violation. Note that external propagators are not used to calculate the effective action, so it is possible to expand perturbatively in the coefficients for Lorentz violation. Problems arise with Hilbert space if one attempts this approximation with external legs.²

Table 1 lists the C, P, and T transformation properties of the field operators in (1), labeled by their associated coefficient. Because QED is itself invariant under both rotation and CPT transformations, the above restriction to linear violation of Lorentz symmetry means that, at this order, the coefficients for Lorentz violation can only receive radiative corrections from coefficients with exactly the same symmetry properties.

Table 1: Discrete-symmetry properties.

	C	P	T	CP	CT	PT	CPT
$c_{00}, (k_F)_{0j0k},$ $c_{jk}, (k_F)_{jklm}$	+	+	+	+	+	+	+
$b_j, g_{j0l}, g_{jk0}, (k_{AF})_j$	+	+	-	+	-	-	-
$b_0, g_{j00}, g_{jkl}, (k_{AF})_0$	+	-	+	-	+	-	-
$c_{0j}, c_{j0}, (k_F)_{0jkl}$	+	-	-	-	-	+	+
a_0, e_0, f_j	-	+	+	-	-	+	-
H_{jk}, d_{0j}, d_{j0}	-	+	-	-	+	-	+
H_{0j}, d_{00}, d_{jk}	-	-	+	+	-	-	+
a_j, e_j, f_0	-	-	-	+	+	+	-

For instance, a_μ has the same symmetries as e_μ and one might expect these parameters to mix. However, in a mass-independent renormalization scheme, such as dimensional regularization,^{11,12} divergent radiative corrections are polynomial in the massive parameters. Thus while a_μ can receive corrections from e_μ , it follows that, on dimensional grounds, e_μ cannot receive corrections from a_μ . In addition, rotational symmetry prevents e_0 and f_j from mixing in this approximation, despite their matching C,P and T transformation properties.

3 Renormalizability at one loop

For a quantum field theory to be renormalizable it is required to have a finite number of divergent, one-particle irreducible Green functions and a sufficient number of parameters in the theory to absorb these divergences. Using the following Feynman rules, deduced from the lagrangian in (1), it can be shown that there are indeed a finite number of superficially divergent Green functions contributing to the effective action.

The propagator for a fermion of momentum p^μ , in the same direction as the charge arrow shown on the diagram, is

$$\longrightarrow = i \frac{(\gamma_\mu p^\mu + m)}{p^2 - m^2}, \quad (3)$$

The Γ_1^μ and M_1 terms give the following propagator insertions

$$\longrightarrow \times \longrightarrow = -iM_1, \quad \longrightarrow \bullet \longrightarrow = i\Gamma_1^\mu p_\mu. \quad (4)$$

The usual photon propagator, with gauge fixing parameter, α ,

$$\mu \text{ } \sim \text{ } \nu = -\frac{i}{p^2} \left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} (1 - \alpha) \right], \quad (5)$$

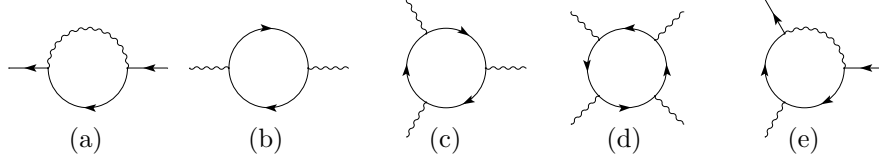


Figure 1: 1PI divergent diagram topologies for QED

receives the following additional insertions, due to $(k_F)_{\kappa\lambda\mu\nu}$ and $(k_{AF})_\mu$,

$$\mu \text{---} \text{---} \bullet \text{---} \nu = -2ip^\alpha p^\beta k_{F\alpha\mu\beta\nu}, \quad \mu \text{---} \text{---} \times \text{---} \nu = 2k_{AF}^\alpha \epsilon_{\alpha\mu\beta\nu} p^\beta. \quad (6)$$

In addition to the usual fermion-photon vertex, there is also a vertex due to Γ_1^μ . These are given in turn by

$$\text{---} \text{---} \text{---} \text{---} \text{---} = -iq\gamma^\mu \quad \text{and} \quad \text{---} \text{---} \bullet \text{---} \text{---} = -iq\Gamma_1^\mu, \quad (7)$$

where q is the fermion charge and μ is the space-time index on the photon line.

The superficial degree of divergence D of a general diagram contributing to the effective action is then

$$D = 4 - \frac{3}{2}E_\psi - E_A - V_{M_1} - V_{AF}, \quad (8)$$

where E_ψ and E_A are the number of external fermion and photon legs and V_{M_1} and V_{AF} are the number of M_1 and $(k_{AF})_\mu$ insertions, respectively. The superficially divergent diagrams have $D \geq 0$ and include the usual divergent diagrams which arise in conventional QED at one loop, (Fig. 1). The remaining one-loop divergent diagrams have the same topologies as Fig. 1 but include exactly one insertion of a Lorentz-violating operator. For example, Fig. 2 shows the set of such diagrams with the same topology as Fig. 1e.

The divergent integrals are regulated using dimensional regularization in $4 - 2\epsilon$ dimensions.^a Working at linear order in the coefficients for Lorentz violation enables these coefficients to be taken outside the integral and ensures that all integrands transform in the conventional manner under particle Lorentz transformations. They can thus be dealt with by conventional techniques and for similar reasons there are no problems performing the Wick rotation.

^aThe analysis has also been performed in the Pauli-Villars scheme and yields equivalent results.

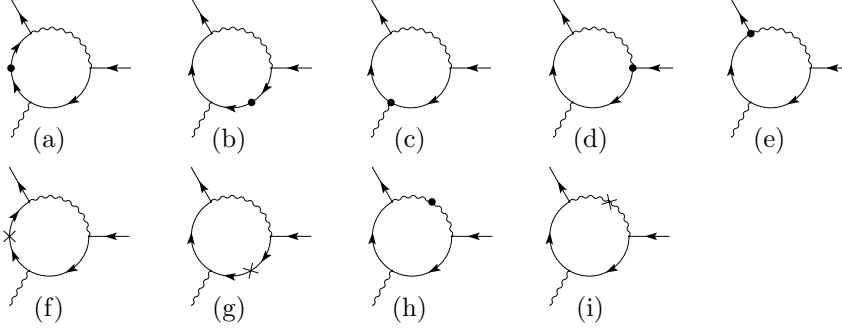


Figure 2: Divergent fermion-photon diagrams in the QED extension

Throughout this work, the naive definition of γ_5 in d dimensions, which anticommutes with all other γ -matrices, is used. This is because we are only interested in the divergent contributions to the effective action and at one loop, this simplification leads only to irrelevant errors in the finite corrections.

Unlike tree-level calculations, loop calculations involve an integration over an infinite range of momenta, but because of the difficulties with stability and microcausality near the Planck scale⁵ the validity of the Feynman rules at this energy scale is unclear. However, it is customary to assume that the low energy physics is not sensitive to the details of the physics at high energy, and therefore it is reasonable to employ the Feynman rules over the entire range of the integration. Further justification of this assumption would be of interest.

In an abelian gauge field theory there can be no divergent three- or four-point photon interactions because they do not correspond to a tree-level interaction. In conventional QED, absence of the three-point radiative corrections is assured by the Furry theorem⁹ which states that there are in fact two nonzero three-point diagrams with opposing charge flow that cancel each other precisely. This depends crucially on the transposition properties of the γ -matrices at the fermion-photon vertex. The QED extension has more complicated γ -matrix structure and an example of a situation arising in this more general case is illustrated in Fig. 3, where there is an insertion of a Lorentz violating operator, Γ_1^ν , at one of the vertices. The sum of these two diagrams is now proportional to $(\Gamma_1^\nu - \tilde{\Gamma}_1^\nu)$, where

$$\tilde{\Gamma}_1^\nu \equiv c^{\mu\nu}\gamma_\mu - d^{\mu\nu}\gamma_5\gamma_\mu - e^\nu - if^\nu\gamma_5 + \frac{1}{2}g^{\lambda\mu\nu}\sigma_{\lambda\mu}. \quad (9)$$

and so the γ_μ terms cancel as in the usual Furry analysis, as do the $\sigma_{\lambda\mu}$ terms, but the I , γ_5 , $\gamma_5\gamma^\mu$ do not. Similar arguments show that the same

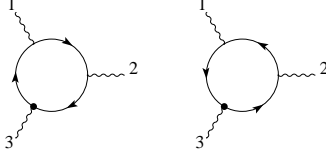


Figure 3: Two contributions to the cubic photon interaction.

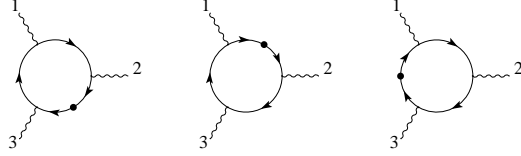


Figure 4: Permutations for the cubic photon vertex.

conclusion is true for Γ_1^μ insertions in propagators, but the opposite is true for M_1 insertions, where it is the γ_μ and $\sigma_{\lambda\mu}$ terms which survive while the others cancel. The analysis is similar for the four-point vertex, but in this case the sum is proportional to $(\Gamma_1^\mu + \tilde{\Gamma}_1^\mu)$, which means that it is now the γ_μ and $\sigma_{\lambda\mu}$ terms which survive and the I , γ_5 , $\gamma_5\gamma^\mu$ which cancel. Likewise for M_1 insertions it is the γ_μ and $\sigma_{\lambda\mu}$ terms which cancel while the others survive. These arguments are applicable to diagrams with any number of photon legs with linear insertions of Lorentz-violating operators.

It follows immediately that there are no corrections to the three-point photon vertex proportional to b_μ , $c_{\mu\nu}$, or $g_{\lambda\mu\nu}$ and no four-point corrections dependent on a_μ , $d_{\mu\nu}$, e_μ , f_μ , or $H_{\mu\nu}$. Other contributions must be explicitly evaluated and there are in fact n of each type for an n -point photon interaction, as illustrated in Fig. 4 for a three-point interaction with Γ_1^μ propagator insertion. The sum of all such terms for a given coefficient is zero, as required for renormalizability.

The remaining one-loop divergences in the theory⁸ must be removed by defining renormalization constants in the usual way,

$$\psi_B = \sqrt{Z_\psi}\psi, \quad A_B^\mu = \sqrt{Z_A}A^\mu, \quad m_B = Z_m m, \quad (10)$$

as well as renormalizing the coefficients for Lorentz violation such that:

$$a_{B\mu} = (Z_a)_\mu^\alpha a_\alpha, \quad b_{B\mu} = (Z_b)_\mu^\alpha b_\alpha, \quad c_{B\mu\nu} = (Z_c)_{\mu\nu}^{\alpha\beta} c_{\alpha\beta} \dots \quad (11)$$

and so on. (Bare parameters are denoted with a subscript B .) These factors can be determined⁸ from the propagator corrections and the conventional QED

fermion-photon vertex alone. They are, however, sufficient to renormalize all of the divergences which appear at one-loop order, including those which arise from the extra corrections to the fermion-photon vertex due to the presence of the Lorentz-violating operators in Eq. (1). This is because the Ward identity of QED, $Z_q\sqrt{Z_A} = 1$, is preserved, meaning that the Lorentz-violating extension of QED preserves gauge invariance and is renormalizable at one-loop.

4 Application of the Renormalization Group

In a renormalizable quantum field theory one can apply the renormalization-group technique to study the evolution of the couplings over a wide range of energies. In the present case we can only be certain that the theory is renormalizable at one-loop order. In the paper by Kostelecký et al.⁸ there is a discussion of the issues concerning the applicability of the usual renormalization-group techniques to the QED extension of Eq. (1). It is argued that it is reasonable to assume that one can apply the usual renormalization-group technique at one-loop in a theory which is multiplicatively renormalizable to that order. Beyond that order there may be nonrenormalizable contributions which would invalidate the assumption of multiplicative renormalizability used to derive the renormalization-group equation. The assumption that the renormalization-group method is applicable at one-loop can be summarized as the assumption that it is reasonable to ignore higher loop effects in a one-loop calculation as long as the couplings which describe the physics in this approximation remain small and any nonrenormalizable couplings remain negligible. Clarification of the extent to which this assumption is justified would be of interest.

Bearing in mind these caveats, it is possible to apply the renormalization-group method to the calculation of the running couplings in a way that is indistinguishable in practice from the usual approach for an all-orders renormalizable quantum field theory. This method relies on a knowledge of the beta functions for the parameters in the theory. In a theory with couplings $\{x_j\}, j = 1, 2, \dots, N$ the beta function⁷ for x_j , is defined as

$$\beta_{x_j} \equiv \mu \frac{dx_j}{d\mu}, \quad (12)$$

where μ is the mass parameter introduced to define the regularization scheme.

In dimensional regularization, it is usual to renormalize each parameter as follows:

$$x_{jB} = \mu^{\rho_{x_j}\epsilon} Z_{x_j} x_j, \quad (13)$$

where the introduction of $\mu^{\rho_{x_i}}$ ensures that the bare parameter, x_{jB} , and its associated renormalized parameter x_j have the same dimension. It is easy to

show that, for the QED extension $\rho_q = 1$ and all other ρ_{x_i} are zero. The usual analysis¹³ then shows that the beta function is given by

$$\beta_{x_j} = \lim_{\epsilon \rightarrow 0} \left[-\rho_{x_j} a_1^j + q \frac{\partial a_1^j}{\partial q} \right], \quad (14)$$

which involves only the simple ϵ -poles, a_1^j , in the renormalization factors Z_{x_j} . Hence the beta functions, and therefore the running of the various parameters in this theory, is determined by the q -dependence of the renormalization factors alone.

For example, the renormalization factor for $d_{\mu\nu}$ is⁸

$$(Z_d)_{\mu\nu}{}^{\alpha\beta} d_{\alpha\beta} = d_{\mu\nu} + \frac{q^2}{12\pi^2\epsilon} (d_{\mu\nu} + d_{\nu\mu}), \quad (15)$$

and hence, using Eq. (14), the beta function is found to be

$$(\beta_d)_{\mu\nu} = \frac{q^2}{6\pi^2} (d_{\mu\nu} + d_{\nu\mu}). \quad (16)$$

From Eq. (12), this can be rewritten as

$$\frac{d}{d \ln \mu} \left[Q^2 (d + d^T) \right] = 0. \quad (17)$$

where

$$Q(\mu) \equiv 1 - \frac{q_0^2}{6\pi^2} \ln \frac{\mu}{\mu_0}, \quad (18)$$

determines the running of q with the scale μ in conventional QED. In particular, with the boundary conditions $x_{j0} \equiv x_j(\mu_0)$, the standard result is $q(\mu)^2 = Q^{-1} q_0^2$ and it follows from Eq. (17), that $d_{\mu\nu}$ runs like Q^{-2} .

In a similar way the running of the other coefficients for Lorentz violation has been found.⁸ All of the mixing is consistent with the predictions made at the end of section 2. The running is determined entirely by $Q(\mu)$ in Eq. (18), but the powers of $Q(\mu)$ involved range from -3 to $9/4$. For example, a_μ and $c_{\mu\nu}$ run as follows

$$\begin{aligned} a_\mu &= a_{0\mu} - m_0 (1 - Q^{9/4}) e_{0\mu}, \\ c_{\mu\nu} &= c_{0\mu\nu} - \frac{1}{3} (1 - Q^{-3}) (c_{0\mu\nu} + c_{0\nu\mu} - (k_F)_{0\mu\alpha\nu}{}^\alpha). \end{aligned} \quad (19)$$

while e_μ does not run at all in this approximation.

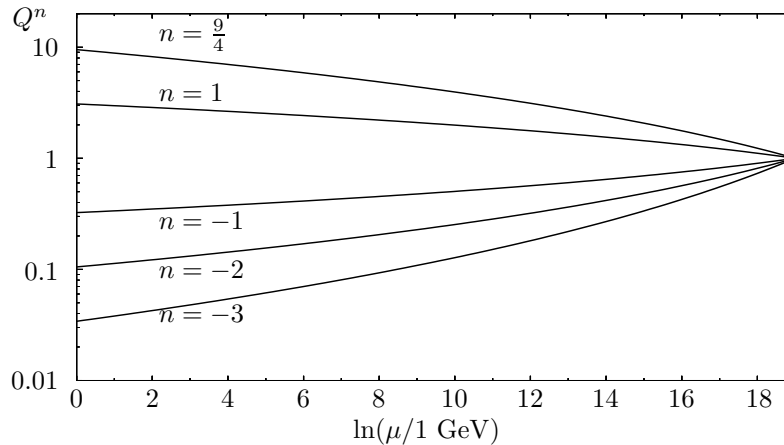


Figure 5: Variation of the function $Q(\mu)^n$.

To apply this analysis to a realistic theory one must include the full standard-model fermion content and possibly allow for the effects of charged scalars as well as the embedding of the U(1) gauge group in a unification group. The precise behavior of the function $Q(\mu)$ is determined by the value of the coefficient of $\ln(\mu/\mu_0)$, which is highly model-dependent. It is therefore difficult to apply the results of these calculations directly to the realistic situation, but one might argue⁸ that a reasonably realistic choice for this coefficient is simply 1. This particular case has been plotted in Fig. 6 to show the behavior of $Q(\mu)^n$ between the weak and Planck scales for various values of n which appear in the running equations for the coefficients for Lorentz violation. The running is far too small to account for the extreme suppression of these coefficients if they are assumed to be $\mathcal{O}(1)$ at the Planck scale. One might expect nonrenormalizable terms, known to be essential to maintain stability and causality near the Planck scale,⁵ to increase the rate of running of the parameters because they introduce negative mass dimension couplings which would be expected to run faster than those considered here.

Figure 6 shows that the variation of $Q(\mu)$ with the scale μ is large enough to produce an appreciable spread at low energies in the values of the various coefficients describing Lorentz violation. In addition, in the full standard-model extension it would be necessary to include the effects of the factors $Q_2(\mu)$ and $Q_3(\mu)$ arising from running of the SU(2) and SU(3) gauge couplings, respectively. Therefore, because of the potential effects of renormalization-group running on the relative sizes of the parameters describing Lorentz violation, this

work illustrates the importance of placing experimental bounds on coefficients from all sectors of the standard-model extension and not simply assuming that they are of the same order of magnitude at the low energy scale.

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